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Wave propagation in viscoelastic wedge with an arbitrary angle peaks

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Abstract. *Modeling of wave propagation in bodies with different geometrical form, is an urgent task. Of particular interest is the construction of the dispersion curves for the deformable wedge considering rheological properties of the material. The aim of this work is a theoretical study of changes in the complex phase velocity of the wave number. The object of this study is deformable wedge. Wedge material is linearly viscoelastic. Boundary value problem for a system of differential equations is solved using the method of lines, which allows you to use the method of orthogonal sweep Godunov and Mueller. The calculation results obtained on the dimensionless quantities. The viscoelastic properties of the material are described by the three-parameter relaxation kernel Koltunov - Ryzhanitsen. For the numerical implementation of the problem, use a tool MAPLE 9.5. The results of calculations are compared with the known data [2], and differ by 6-20%. Accounting for the viscoelastic properties of the material of the wedge increases the real part of the wave propagation velocity is 10-15%, and also allows to evaluate the damping of the system as a whole. It is found that for small wedge angles allowed the use of the simplified theory of Kirchhoff - Love and Timoshenko throughout the wavelength range.*
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I. Introduction

In [1,2,3] the propagation of waves in viscoelastic extended plates and layers of variable thickness. In these studies found that the role of boundaries in determining the structure of the wave field, the spectrum of the natural frequencies and their own forms, revealed in a series of simple tasks, and change the border accompanied by consistently increasing difficulties. It also discusses the emergence of local singularities in the wave fields.

In this paper, in contrast to these considered the wave propagation along the z axis in an infinite viscoelastic cylinder with a radial crack, which is a wedge at an angle $|\varphi|\langle 90^0$.

II. Statement of the problem and their solutions

The basic equations of motion of a deformable cylinder (of radius R) with a radial crack, which when $\varphi=|\varphi_0|\ \langle 90^0$ case describes a wedge. They are set with three groups of relations. The system of equations of motion of the wedge in a cylindrical coordinate system (r,φ,z) takes the form

$$\rho\frac{\partial u_r}{dt^2}=\frac{\partial\sigma_{rr}}{dr}+\frac{\sigma_{rr}-\sigma_{r\varphi}}{r}+\frac{1}{r}\frac{\partial\sigma_{r\varphi}}{\partial\varphi}+\frac{\partial\sigma_{rz}}{\partial z};$$
$$\rho\frac{\partial^2 u_\varphi}{\partial t^2}=\frac{1}{r}\frac{\partial\sigma_{\varphi\varphi}}{\partial\varphi}+\frac{2\sigma_{r\varphi}}{r}+\frac{\partial\sigma_{r\varphi}}{\partial r}+\frac{\partial\sigma_{z\varphi}}{\partial z};\quad (1)$$
$$\rho\frac{\partial^2 u_z}{\partial t^2}=\frac{\partial\sigma_{zz}}{\partial z}+\frac{\partial\sigma_{rz}}{\partial r}+\frac{\sigma_{zz}}{r}+\frac{1}{r}\frac{\partial\sigma_{z\varphi}}{\partial\varphi}.$$

Here

$$\varepsilon_{rr}=\frac{\partial u_r}{\partial r};\qquad \varepsilon_{zz}=\frac{\partial u_z}{\partial z};\quad \varepsilon_{\varphi\varphi}=\frac{1}{r}\frac{\partial u_r}{\partial\varphi}+\frac{u_r}{r};\qquad (2)$$

$$\varepsilon_{r\varphi} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r}\right);\ \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right);$$

$$\varepsilon_{\varphi z} = \frac{1}{2}\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right);$$

$$\sigma_{rr} = \tilde{\lambda}\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\tilde{\mu}\frac{\partial u_r}{\partial r};$$

$$\sigma_{r\varphi} = 2\tilde{\mu}\varepsilon_{r\varphi} = \tilde{\mu}\left(\frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r}\right);$$

$$\sigma_{rz} = 2\tilde{\mu}\varepsilon_{rz} = \tilde{\mu}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right); \qquad (3)$$

$$\sigma_{\varphi\varphi} = \tilde{\lambda}\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\tilde{\mu}\left(\frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r}\right);$$

$$\sigma_{\varphi z} = \tilde{\mu}\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right);$$

$$\sigma_{zz} = \tilde{\lambda}\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\tilde{\mu}\frac{\partial u_z}{\partial z},$$

where

$$\tilde{\lambda}\ f(t) = \lambda_0\left[f(t) - \int_0^t R_\lambda(t-\tau) f(\tau) d\tau\right], \qquad (4)$$

$$\tilde{\mu}\ f(t) = \mu_0\left[f(t) - \int_0^t R_\mu(t-\tau) f(\tau) d\tau\right];$$

$f(t)$ - a function; ρ - density materials, $R_\mu(t-\tau)$ and $R_\lambda(t-\tau)$ - the core relaxation, λ_o , μ_0 - the instantaneous modulus of elasticity of a viscoelastic medium, $\vec{u}\left(u_r, u_\varphi, u_z\right)$ is the vector displacement which depends $\sigma_{rr}, \sigma_{r\varphi}, \sigma_{rz}, \sigma_{\varphi\varphi}, \sigma_{\varphi z}, \sigma_{zz}$ - respectively, the components of the stress tensor; $\varepsilon_{rr}, \varepsilon_{r\varphi}, \varepsilon_{rz}, \varepsilon_{\varphi\varphi}, \varepsilon_{\varphi z}, \varepsilon_{zz}$ - respectively, the components of the strain tensor. Equation (4) after the application of the method of freezing [4] takes the following form:

$$\bar{\lambda}\ f(t) = \lambda\left[1 - \Gamma_\lambda^C(\omega_R) - i\Gamma_\lambda^S(\omega_R)\right] f(t),\quad \bar{\mu}\ f(t) = \mu_m\left[1 - \Gamma_\mu^C(\omega_R) - i\Gamma_\mu^S(\omega_R)\right] f(t),$$

where $\Gamma_\lambda^C(\omega_R) = \int_0^\infty R_\lambda(\tau)\cos\omega_R\tau\, d\tau$, $\Gamma_\lambda^S(\omega_R) = \int_0^\infty R_\lambda(\tau)\sin\omega_R\tau\, d\tau$,

$\Gamma_\mu^C(\omega_R) = \int_0^\infty R_\mu(\tau)\cos\omega_R\tau\, d\tau$, $\Gamma_\mu^S(\omega_R) = \int_0^\infty R_\mu(\tau)\sin\omega_R\tau\, d\tau$ - , respectively, the cosine and sine Fourier transforms; ω_R - the real part of the complex frequency ($\omega = \omega_R + i\omega_I$); ρ - density; $R_\lambda(t)$ and $R_\mu(t)$ respectively relaxation kernel material. Relation (1), (2), (3) after algebraic manipulations are identical to the system of six differential equations with complex coefficients are solved for the first derivative with respect to the radial coordinate

$$\begin{cases} \dfrac{\partial u_r}{\partial r} = \dfrac{1}{K}\sigma_{rr} - \dfrac{\overline{\lambda}}{K}\left(\dfrac{1}{r}\dfrac{\partial u_\varphi}{\partial \varphi} + \dfrac{u_r}{r} + \dfrac{\partial u_z}{\partial z}\right); \\ \dfrac{\partial u_\varphi}{\partial r} = \dfrac{1}{\overline{\mu}}\sigma_{r\varphi} - \dfrac{1}{r}\left(\dfrac{\partial u_r}{\partial \varphi} - u_\varphi\right); \\ \dfrac{\partial u_z}{\partial r} = \dfrac{1}{\overline{\mu}}\sigma_{rz} - \dfrac{\partial u_r}{\partial z}; \\ \dfrac{\partial \sigma_{rr}}{\partial r} = \rho\dfrac{\partial^2 u_r}{\partial t^2} - \dfrac{\tilde{A}}{r} - \dfrac{1}{r}\dfrac{\partial \sigma_{r\varphi}}{\partial \varphi} - \dfrac{\partial \sigma_{rz}}{\partial z}; \\ \dfrac{\partial \sigma_{r\varphi}}{\partial r} = \rho\dfrac{\partial^2 u_\varphi}{\partial t^2} - \dfrac{1}{r}\dfrac{\partial}{\partial \varphi}\left[\sigma_{rr} - \tilde{A}\right] - \dfrac{2\sigma_{r\varphi}}{r} - \dfrac{\partial}{\partial z}\tilde{B}; \\ \dfrac{\partial \sigma_{rz}}{\partial r} = \rho\dfrac{\partial^2 u_z}{\partial t^2} - \dfrac{\partial}{\partial z}\left[\sigma_{rr} - 2\overline{\mu}\left(\dfrac{\partial u_r}{\partial r} - \dfrac{\partial u_z}{\partial z}\right)\right] - \dfrac{\sigma_{rz}}{r} - \dfrac{1}{r}\dfrac{\partial}{\partial \varphi}\tilde{B}; \end{cases} \quad (5)$$

where we have introduced the notation

$$\tilde{A} = 2\overline{\mu}\left[\frac{\partial u_r}{\partial r} - \frac{1}{r}\left(\frac{\partial u_\varphi}{\partial \varphi} + u_r\right)\right]; \quad \tilde{B} = \overline{\mu}\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right).$$

The boundary conditions are specified as:

$$r = r_0 \to 0 \text{ и } R: \ \sigma_{rz} = \sigma_{rr} = \sigma_{r\varphi} = 0$$

$$\varphi = -\frac{\varphi_0}{2}, \frac{\varphi_0}{2}; \quad \sigma_{\varphi\varphi} = \sigma_{\varphi r} = \sigma_{\varphi z} = 0 \quad (6)$$

Periodicity conditions allow to eliminate the dependence of the basic unknowns of time and axial coordinate z with the following change of variables:

$$u_r = w(r)\cos\frac{\varphi}{2}e^{i\kappa(z-ct)}; \quad u_\varphi = v(r)\sin\frac{\varphi}{2}e^{i\kappa(z-ct)}; u_z = u(r)\cos\frac{\varphi}{2}e^{i\kappa(z-ct)};$$
$$\sigma_{rr} = \sigma(r)\cos\frac{\varphi}{2}e^{i\kappa(z-ct)}; \sigma_{r\varphi} = \tau_\varphi(r)\sin\frac{\varphi}{2}e^{i\kappa(z-ct)}; \quad \sigma_{rz} = \tau_z(r)\cos\frac{\varphi}{2}e^{i\kappa(z-ct)}, \quad (7)$$

where $W(r), v(r), u(r), \sigma(r), \tau_\varphi(r), \tau_z(r)$ - amplitudes of fluctuations which are function of radial coordinate; κ -wave number; $c = c_R + ic_I$ -complex phase speed; $\omega = \omega_R + i\omega_I$ -complex frequency. Under condition of (6) division of variables *r* and *φ*, it is impossible. Taking into account (7) system of the equations (5) becomes:

$$\begin{cases} w' = \dfrac{\sigma}{K} - \dfrac{\overline{\lambda}}{K}\left(ku + \dfrac{1}{r}\left(w + \dfrac{\partial v}{\partial \varphi}\right)\right) \\ v' = \dfrac{\tau_\varphi}{\overline{\mu}} + \dfrac{1}{r}\left(v - \dfrac{\partial w}{\partial \varphi}\right) \\ u' = \dfrac{\tau_z}{\overline{\mu}} + kw \\ \sigma' = -\omega^2 \rho w + \dfrac{1}{r}\left(A - \dfrac{\partial \tau_\varphi}{\partial \varphi}\right) - k\tau_z \\ \tau'_\varphi = -\omega^2 \rho v - \dfrac{1}{r}\left(\dfrac{\partial(A+\sigma)}{\partial \varphi} + 2\tau_\varphi\right) - kB \\ \tau'_z = -\omega^2 \rho u - \dfrac{1}{r}\left(\dfrac{\partial B}{\partial \varphi} + \tau_z\right) + k(\sigma + 2\mu(ku - w')) \end{cases} \quad (8)$$

where

$$A = 2\overline{\mu}\left(\frac{1}{2}\left(\frac{\partial v}{\partial \varphi} + w\right) - w'\right) \quad B = \overline{\mu}\left(\frac{1}{r}\frac{\partial u}{\partial \varphi} - kv\right)$$

Similarly transformed boundary conditions (6)

$$r = 0, R: \quad \sigma = \tau_\varphi = \tau_z = 0 \ . \quad (9)$$

Thus, it is formulated a spectral boundary problem (8), (9), describing distribution of harmonious waves in an infinite viscoelastic wedge with any corner of top.

As an example of a viscoelastic material we will accept the three-parametrical kernel of a relaxation $R_\lambda(t) = R_\mu(t) = Ae^{-\beta t}/t^{1-\alpha}$ possessing a weak singularity.

The boundary problem for system of the differential equations in private derivatives (8), by means of a method of straight lines, allows to use a method of orthogonal pro-race[5]. According to a method of straight lines the rectangular range of definition of function of the main unknown becomes covered by the straight lines which parallel to an axis r and are evenly costing from each other.

The decision we find only for these straight lines, and the derivative in the direction *φ*, is replaced with approximate final differences. Used approximating formulas of the second order for the first and second derivative look like:

$$y_{i,\varphi} \cong \frac{y_{i+1} - y_{i-1}}{2\Delta} \cong \frac{-3y_i + 4y_{i+1} - y_{i+2}}{2\Delta} \cong \frac{3y_i - 4y_{i-1} + y_{i-2}}{2\Delta}$$

$$y''_{i,\varphi} \cong \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta^2} \quad (10)$$

where i changes from 0 to $N+1\left(i = \overline{0, N+1}\right)$, y - a projection of unknown function to a straight line with number i; Δ - a splitting step on coordinate φ.

As a result of digitization the vector of the main unknown of the general dimension 6N can be written down in a look:

$$Y = (\{w_i\}, \{v_i\}, \{u_i\}, \{\sigma_{ri}\}, \{\tau_{\varphi i}\}, \{\tau_{zi}\})^T \quad i = \overline{1, N} \quad (11)$$

The central differences (10), (11) are used for internal straight lines (1 <i <N), the left and right differences (10), (11) allow to consider boundary conditions on φ. In the first case a derivative on φ in the right parts of system of the equations (8) it is expressed on formulas:

1 <i<N

$$w_{i,\varphi} = (w_{i+1} - w_{i-1})/2\Delta$$
$$u_{i,\varphi} = (u_{i+1} - u_{i-1})/2\Delta$$
$$v_{i,\varphi} = (v_{i+1} - v_{i-1})/2\Delta \quad (12)$$

$$\tau_{\varphi_i,\varphi} = \left(\tau_{\varphi(i+1)} - \tau_{\varphi(i-1)}\right)/2\Delta$$

$$\sigma_{\varphi_i,\varphi} = a(\sigma_{i+1} - \sigma_{i-1})\Big/2\Delta + \frac{b}{r}\left[(v_{i+1} - 2v_i + v_{i-1})/\Delta^2 + w_{i,\varphi}\right] + cku_{i,\varphi}$$

$$B_i = (u_{i+1} - 2u_i + u_{i-1})/\Delta^2/k - kv_{i,\varphi}$$

Boundary conditions at $\varphi = -\frac{\varphi_0}{2}$ it is considered in the equations, the corresponding straight lines of *i=1*. For the main unknown who are not entering into boundary conditions of $\boldsymbol{w}_i$, $\boldsymbol{v}_i$, $\boldsymbol{u}_i$ the right differences (10) are used:

$$\begin{aligned} w_{i,\varphi} &= (-3w_1 + 4w_2 - w_3)/2\Delta \\ v_{i,\varphi} &= (-3v_1 + 4v_2 - v_3)/2\Delta \\ u_{i,\varphi} &= (-3u_1 + 4u_2 - u_3)/2\Delta \end{aligned} \qquad (13)$$

For a variable τ_φ , σ_φ , *B* by means of the central difference

$$\tau_{\varphi_i,\varphi} \cong \left(\tau_{\varphi_2} - \tau_{\varphi_0}\right)/2\Delta = -\tau_{\varphi_2}/2\Delta \qquad (14)$$

$$\sigma_{\varphi_1,\varphi} \cong \left(\sigma_{\varphi_2} - \sigma_{\varphi_0}\right)/2\Delta = \sigma_{\varphi_2}\Big/2\Delta = \left(a\sigma_{r_2} + \frac{b}{r}\left[(v_3 - v_1)/2\Delta + w_2\right] - cku_2\right)\Big/2\Delta$$

$$B_{1,\varphi} \cong (B_2 - B_0)/2\Delta = B_2/2\Delta = \left[(u_3 - u_1)/2\Delta/r - kv_2\right]/2\Delta \ .$$

Derivatives for a straight line with the number *i=N*, considering boundary conditions are similarly represented at $\varphi = \frac{\varphi_0}{2}$. The unique difference consists in replacement of the right final differences with the left: *i=N*

$$w_{i,\varphi} = (3w_N - 4w_{N-1} + w_{N-2})/2\Delta, \quad v_{i,\varphi} = (3v_N - \ldots)/2\Delta$$

$$U_{i,\varphi} = (U_{i+1} - U_{i-1})/2\Delta, \quad u_{i,\varphi} = (3u_N - \ldots)/2\Delta \qquad (15)$$

$$\tau_{\varphi,\varphi} = -\tau_{\varphi(N-1)}/2\Delta$$

$$\sigma_{i,\varphi} = \left(a\sigma_{N-1} + \frac{b}{r}\left[(v_N - v_{N-2})/2\Delta + w_{N-1}\right] + cku_{N-1}\right)\Big/2\Delta = -\frac{\sigma_{N-1}}{2\Delta}$$

$$B_{i,\varphi} = -\left[(u_N - u_{N-2})/2\Delta/r - kv_{N-1}\right]\Big/2\Delta = -\frac{B_{N-1}}{2\Delta}$$

Thus, an initial spectral task (8), (9) by means of coordinate digitization φ on a method of straight lines it is reduced to an initial task (11). To the solution of this task we will apply a method of orthogonal pro-race.

III. Numerical results

Stretch value in the formulation of the problem are selected so that the shear velocity C_s, the density ρ and the outer radius *R* are the single value. And also $A = 0{,}048; \quad \beta = 0{,}05; \quad \alpha = 0{,}1$. For the numerical implementation of the problem, use a tool MAPLE 9.5.

In the table 1 ($R_\lambda(t) = R_\mu(t) = 0$) limiting values of phase speed of the first edged fashion depending on a wedge corner are given in top (in terms of thickness of a wedge in h_2 basis) (column 2), found for a material with Poisson factor υ = 0,25 according to the theory of plates Kirchhoff - Love (column 3), Timoshenko - (column 4). Within the design procedure of a three-dimensional wedge stated in this article (column 5-6) and on a formula $C_0 = C_R \ \sin(m\varphi)$ [2], *m = 1, 2, ..., mφ < 90 °* (column 7). The column 5 corresponds to calculation option with three internal straight lines (N = 3) and boundary conditions (8), the column 6 corresponds to boundary conditions:

$$\varphi = -\frac{\varphi_0}{2}; \sigma_{\varphi\varphi} = \sigma_{\varphi r} = \sigma_{\varphi z} = 0; \varphi = 0; u_r = u_z = \sigma_{\varphi\varphi} = 0$$

at the same quantity of straight lines. According to numerical results, and given in table 1, options of calculation for Kirchhoff – Love method, Timoshenko method and the three-dimensional theory will be coordinated among themselves within 7 % for wedge corners with a thickness in the basis h_2 which is not exceeding 0,5 (a wedge corner $\varphi_0 = 28°$). Thus, unlike wave guides with rectangular section in wedge-shaped wave guides with rather small corner of a wedge in the analysis of dispersive dependences of the first fashion is admissible to use the theory of plates Kirchhoff - Love. Established fact that the phenomenon is due to the localization of the waveform near the acute angle of the wedge, as described in [3].

Table 1: Calculation methods for the Kirchhoff - Love, Timoshenko and three-dimensional theory.

h_2	φ_0	by the method of the Kirchhoff-Love	Timoshenko method	the method for calculating the three-dimensional wedge (1)	the method for calculating the three-dimensional wedge (2)	at work [2]
0,2	11^0	0,2	0,196	-	-	0,182
0,3	17^0	0,3	0,286	0,308	0,298	0,276
0,5	28^0	0,5	0,442	0,475	0,462	0,433
0,7	38^0	0,7	0,563	0,605	0,592	0,574
1	53^0	1	0,691	0,741	0,729	0,736
2	90^0	2	0,864	0,908	-	0,92

On the basis of the received results the following conclusions are drawn:

- results of calculation of the limiting speed($c = C_R$) of propagation of the first mode tapered waveguide on the theory of plate Kirchhoff – Love[2] and the dynamic theory of elasticity does not differ by more than 6% for the edge of the wedge angle not exceeding 28°. At $28^0 < \varphi < 90^0$ calculation results differ up to 20%.

- account the viscoelastic properties of the material of the wedge increases the real part of the wave propagation velocity of 10-15%, as well as to evaluate the damping capacity of the system as a whole.

Thus, for small wedge angles allowed the use of the simplified theory of Kirchhoff - Love and Timoshenko throughout the wavelength range.

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